Benchmarking Low-Volatility Strategies

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Abstract
In this paper we discuss the benchmarking of low-volatility investment strategies, which are designed to benefit from the empirical result that low-risk stocks tend to earn high risk-adjusted returns. Although the minimum-variance portfolio of Markowitz is the ultimate low-volatility portfolio, we argue that it is not a suitable benchmark, as it can only be determined with hindsight. This problem is overcome by investable minimum-variance strategies, but because various approaches are equally effective at minimizing volatility it is ambiguous to elevate the status of any one particular approach to benchmark. As an example we discuss the recently introduced MSCI Minimum Volatility indices and conclude that these essentially resemble active low-volatility investment strategies themselves, rather than a natural benchmark for such strategies. In order to avoid these issues, we recommend to simply benchmark low-volatility managers against the capitalization-weighted market portfolio, using risk-adjusted performance metrics such as Sharpe ratio or Jensen’s alpha.
1. Introduction

Low-risk stocks tend to earn high risk-adjusted returns. This stylized fact was first established by Black, Jensen and Scholes (1972) and Fama and MacBeth (1973), whose empirical tests of the Capital Asset Pricing Model (CAPM) showed that the relation between beta and return in the U.S. stock market is flatter than predicted by theory. This effect does not appear to have weakened over time, does not depend critically on the choice of risk measure (e.g. volatility instead of beta) or risk calculation method (e.g. 3- or 5-year trailing data) and has also been confirmed for international markets; see Fama and French (1992), Black (1993), Falkenstein (1994), Blitz and Van Vliet (2007) and Baker, Bradley, and Wurgler (2010). These studies suggest that the empirical relation between volatility and return in the equity market is flat or negative, a phenomenon which throughout this paper we will refer to as “the volatility effect”.

The studies above consider quantile portfolios based on a ranking of the universe of stocks on risk measures such as past volatility or beta. Further support for the existence of a volatility effect is provided by a related stream of literature, which uses mean/variance optimization to determine the portfolio with minimum past volatility, see for example Haugen and Baker (1991) and Clarke, de Silva and Thorley (2006).¹ Although these authors use a more sophisticated technique to construct low-volatility portfolios (for example, optimization also takes into account correlation effects), their conclusions are similar: portfolio volatility can be reduced significantly without negative implications for expected returns.

To summarize, the literature reports a persistent volatility effect, which appears to be robust to research design choices with regard to sample period, universe and portfolio construction methodology. Attracted by these findings, an increasing number of investors have adopted low-volatility investment strategies in recent years. One of the questions which arises as a result of this trend is how to evaluate the performance of low-volatility managers in practice.

¹ Examples of other low-volatility portfolio construction techniques that have been proposed are “equally-weighted risk contribution” portfolios, see Maillard, Roncalli and Teiletche (2010), and “most diversified” portfolios, see Choueifaty and Coignard (2008).
The first thought that may come to mind is to simply benchmark low-volatility managers against an appropriate low-volatility index. However, we argue that this is not as straightforward as it may seem. To begin with, a benchmark should represent an investable alternative. This disqualifies the unique minimum-variance portfolio that exists in theory, as it is not observable \textit{ex ante}. Investable approximations to the true minimum-variance portfolio may be considered instead, but here the problem is that various approaches appear to be equally valid, implying that it is arbitrary to elevate the status of any one particular approach to benchmark for all low-volatility strategies. As an example we discuss the recently introduced MSCI Minimum Volatility indices, concluding that these essentially resemble active low-volatility investment strategies themselves, rather than providing a benchmark for the entire class of low-volatility strategies. Although a comparison between the performance of a low-volatility manager and a minimum-volatility index may be informative, a more general and universally applicable method is to be preferred. We propose to simply benchmark low-volatility managers against the capitalization-weighted market portfolio, using established risk-adjusted performance metrics such as Sharpe ratio or Jensen’s alpha.

2. A unique low-volatility benchmark does not exist

As it is common practice to benchmark a value manager against a value index, a small-cap manager against a small-cap index and so on, a natural idea would be to benchmark a low-volatility manager against an appropriate low-volatility index. However, this is not as straightforward as it may seem. For example, it raises questions such as how an appropriate low-volatility index should be defined, whether it is fair to evaluate all low-volatility managers against the same index and which specific performance metric should be used.

2.1 A unique minimum-volatility portfolio exists only in theory

At first sight, theory may appear to provide the perfect benchmark for low-volatility managers, namely the minimum-variance portfolio of Markowitz (1952).
The minimum-variance portfolio is defined as the portfolio at the far left of the efficient frontier in risk/return space and is therefore, by definition, the ultimate low-volatility portfolio. Unfortunately, Markowitz’ minimum-variance portfolio is a theoretical construct, which should not be confused with a practically applicable benchmark that can be used to evaluate the performance of low-volatility managers. Calculating the Markowitz minimum-variance portfolio requires the covariance matrix of stock returns, but the problem is that the prevailing covariance matrix at any given point in time is not observable. Of course one can try to estimate the covariance matrix, using historical market data, and next derive a proxy for the true minimum-variance portfolio based on this estimated covariance matrix. However, this is a subjective approach, with an unlimited number of possible outcomes. The unique, true minimum-variance portfolio in the spirit of Markowitz’ theory can only be determined ex post\textsuperscript{2} and therefore does not qualify as an investable alternative. This is a fundamental difference with benchmarks that are commonly used in practice, such as the capitalization-weighted market index, value indices or small-cap indices, all of which can be identified without ambiguity ex ante. We conclude that Markowitz’ theoretical minimum-variance portfolio is not an appropriate tool for benchmarking low-volatility managers.

2.2 Minimum-volatility indices

Although the true minimum-volatility portfolio is unobservable, it might be possible to derive a close approximation in practice that can be used to benchmark low-volatility managers. However, the problem here is that various approaches which have been proposed in the literature appear to be equally effective at approximating the true minimum-volatility portfolio. For example, the literature on minimum-volatility portfolios shows volatility reductions of around 30% (e.g. see Clarke, de Silva and Thorley, 2006), but similar results are obtained for quantile portfolios based on ranking stocks on past volatility or beta.

\textsuperscript{2} This actually still requires some assumptions, e.g. the relevant time period and the way to calculate the covariance matrix (e.g. data frequency).
For example, Black (1993) considers a broad sample of U.S. stocks over the 1931-1991 period and every month ranks these stocks on their past beta. He finds that the \textit{ex post} volatility of the decile portfolio consisting of the stocks with the lowest past beta is 33\% lower than the volatility of the market portfolio. Blitz and van Vliet (2007) rank global stocks over the 1986-2006 period on their past volatility and also find that the resulting top decile portfolio reduces volatility by 33\%. In short, the literature shows that various portfolios, constructed in different manners, succeed in reducing volatility by around one-third \textit{vis-à-vis} the capitalization-weighted market portfolio. Although the long-term volatility reduction achieved by these various approaches is similar, the differences in portfolio composition may result in return divergences in the short run. We therefore conclude that it is ambiguous to elevate the status of any one particular approach to benchmark for low-volatility investment strategies in general.

3. An example: the MSCI Minimum Volatility index series

In April 2008, MSCI launched its so-called “Minimum Volatility” (MV) index series, available for various geographic markets. These indices reflect portfolios that are mechanically optimized for minimum volatility using the MSCI Barra risk model and a fixed set of restrictions. In their press release, MSCI states that “\textit{research shows that the simulated historical performance of the MSCI World Minimum Volatility Index was more than 30\% less volatile than the MSCI World Index over the period December 1998 to December 2007}”. The MV indices aim to offer a better risk/return profile than market capitalization weighted indices by capitalizing on the volatility effect, and are inspired by the stream of literature which also uses optimization techniques to construct minimum-volatility portfolios (see before).

The MSCI MV indices might be considered an appropriate benchmark for low-volatility strategies for several reasons. First, they are constructed in an entirely mechanical, rules-based fashion. Second, they have modest turnover,
which fits with the notion of a passive index. Third, they can be tracked at low costs by passive managers, e.g. to construct ETFs. In these aspects, the MSCI MV indices are similar to other indices that are widely used for benchmarking purposes. Despite these features, however, we argue that the MSCI MV indices do not represent an unambiguous passive benchmark for active low-volatility managers, but are essentially an active investment strategy themselves.

First, as discussed in the previous section, the approach of the MSCI MV indices does not appear to be more effective at reducing volatility than alternative approaches, such as simply ranking stocks on their past beta or volatility. Second, the MSCI MV indices lack transparency and rely on a large number of assumptions. For example, whereas stock weights in the capitalization-weighted market portfolio are simply proportional to their market capitalizations, the composition of MSCI MV indices is based on a complex optimization algorithm which involves a sophisticated, proprietary risk model. The MSCI MV indices also require a large number of assumptions, e.g. with respect to the rebalancing frequency, constraints on the maximum and minimum weight of individual stocks, countries and sectors, constraints on exposures to the Barra risk indices and constraints on turnover. A large number of subjective assumptions and parameter choices is a typical feature of active investment strategies. From a passive strategy, on the other hand, one would expect little or no subjectivity to be involved. Third, the constraints that are imposed on turnover result in path-dependence of the MSCI MV indices. This means that the composition of an

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3 The MSCI MV indices are rebalanced semi-annually with a 10% constraint on one-way turnover, implying a maximum one-way turnover of 20% per annum.

4 Moreover, the ranking approach is actually characterized by lower complexity and higher transparency, making it more suitable as a basis for constructing an index.

5 MSCI also assumes a U.S. dollar base currency when constructing their MV indices. This makes stocks that trade in other currencies, e.g. the euro or Japanese yen, less attractive, because exchange rate fluctuations tend to add to volatility. For investors with a U.S. dollar base currency perspective this may make perfect sense, but for investors with a different base currency it does not. Ideally, these investors should have their own MSCI MV indices, optimized under different base-currency assumptions. This is unlike conventional passive indices, for which the base currency assumption does not affect portfolio composition.

6 Imposing turnover constraints is essential in order to prevent turnover from exploding. For a U.S. equity minimum-variance strategy in the same spirit as the MSCI approach, Clarke, de Silva and Thorley (2006) report a turnover of 11.9% per month, or 143% a year, in case of monthly rebalancing without any constraints on turnover. The good news is that these authors also find
MSCI MV index today depends on its composition in the previous period, which in turn depends on the composition in the period before that, all the way back to the inception date of the index. In other words, the MSCI MV indices are not designed to reflect the optimal minimum-volatility portfolio at each point in time, e.g. for investors who would like to start investing in a minimum-volatility equity strategy from scratch.

4. Benchmarking against the capitalization-weighted index
An alternative to benchmarking low-volatility managers against a low-volatility index is to simply benchmark them against the capitalization-weighted market index. In this case it is important to realize that a straight comparison of returns is not appropriate, given that low-volatility strategies tend to exhibit significantly lower risk (volatility, beta). This problem can be addressed by using a performance evaluation measure that adjusts returns for the level of risk involved. The choice for a specific risk-adjusted performance metric should depend on how one defines risk. If total (systematic and idiosyncratic) volatility is considered to be the relevant risk measure, low-volatility investment strategies can be evaluated against the capitalization-weighted market portfolio using the Sharpe ratio. In case beta (i.e. systematic risk only) is considered to be the relevant risk measure, CAPM alpha or Jensen’s alpha can be used instead. The Sharpe ratio tends to be a bit more conservative, but can be hard to interpret if average returns are negative. Alpha has the advantage of always being directly interpretable: a higher alpha is always better than a lower alpha. Not surprisingly, the Sharpe ratio and Jensen’s alpha are both frequently used metrics in the literature on the low-volatility effect.

In order to illustrate how different ways of benchmarking can lead to different conclusions regarding the performance of low-volatility strategies, we consider the stylized example shown in Exhibit 1. The figure depicts the
capitalization-weighted index, a minimum-volatility index and several hypothetical actively managed low-volatility strategies (A, B, C and D) in risk/return space, where volatility is assumed to be the relevant risk measure. Using the Sharpe ratio as an evaluation measure, portfolio A is the best alternative in this example. Although it has the same return as the MV index, it exhibits lower risk, resulting in a better Sharpe ratio. In fact, portfolio A comes closer to the true minimum-volatility portfolio than the MV index, illustrating our point that, similar to other active low-volatility strategies, the MV index is merely an attempt, and not necessarily the most successful one, to approximate the true, unobservable minimum-volatility portfolio. Alternatively, if the MV index is used as benchmark, portfolio B is identified as best, as it is the only portfolio which beats the MV index in terms of return, while having the same risk. However, this ignores the fact that the return enhancement of portfolio B adds less to its Sharpe ratio than the risk reduction achieved by portfolio A. Portfolio C has the same return as the MV index and the market capitalization weighted index, similar to portfolio A, but is inferior to portfolios A, B and the MV index when taking into account its higher risk. However, in comparison to the capitalization-weighted index portfolio, C is still a superior alternative. This portfolio also serves as an example which illustrates that for a manager with a volatility in between the capitalization-weighted index and minimum-volatility approaches, a comparison of raw returns with either alternative is inappropriate. Finally, portfolio D underperforms in comparison with the MV index and the capitalization-weighted index, but, like portfolio C, still offers a superior risk-adjusted performance compared with the capitalization-weighted index.

5. Summary

An increasing number of investors are adopting low-volatility investment strategies designed to benefit from the empirical result that low-risk stocks offer high risk-adjusted returns. In this paper we have discussed how the performance of low-volatility managers can be evaluated in practice. At first sight the use of a low-volatility index for benchmarking purposes may seem like an obvious choice,
but this approach turns out to be problematic upon closer examination. We first argued that the unique minimum-variance portfolio from Markowitz' portfolio theory is not a useful alternative, because of the fact that it is not observable in practice. By using predictions of future risk, typically derived from past data, it is possible to create practical approximations of the theoretical minimum-volatility portfolio. However, the literature indicates that various approaches are equally effective, reducing volatility by up to one-third, which implies that it is quite arbitrary to elevate the status of one of these approaches to benchmark for all low-volatility approaches.

As an example we discussed the recently introduced MSCI Minimum Volatility index series, arguing that, rather than being a passive benchmark alternative for active low-volatility managers, these indices are essentially active low-volatility investment strategies themselves. First, because the literature shows that a more simple and transparent quantile portfolio approach is at least as effective at reducing volatility as the mean/variance optimization approach used by the MSCI MV indices. Second, because the index construction methodology relies on a large number of subjective assumptions. And, third, because of the path-dependent nature of the MSCI MV indices.

Still, in certain cases, it can make sense to evaluate the performance of a low-volatility manager against a minimum-volatility index such as those of MSCI. A more robust and generally applicable alternative, however, is to simply benchmark low-volatility strategies against the capitalization-weighted market portfolio, using risk-adjusted performance metrics such as Sharpe ratio or Jensen’s alpha. This approach also recognizes that, at the end of the day, low-volatility investing is not primarily aimed at beating a certain low-volatility index, but at establishing a risk/return profile superior to a passive investment in the capitalization-weighted market index.

References


Exhibit 1: Stylized example

The figure illustrates a stylized example of the volatility and return of the capitalization-weighted market index, a minimum-volatility weighted index, and four different low volatility managers. Low-volatility manager B outperforms the minimum volatility index, while manager D underperforms this index. Low-volatility manager A reduces risk further as compared to the minimum-volatility index, whereas manager C adds risk. The solid line depicts the capital market line and the dotted line represents the maximum Sharpe ratio portfolio, which intersects with A.
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