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The Volatility Effect

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The Volatility Effect

Lower risk without lower return.

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fficient markets theory has been challenged by the finding that relatively simple investment strategies are found to generate statistically significantly higher returns than the market portfolio. Well-known examples are value, size, and momentum strategies, whose return premiums have been documented in U.S. and international stock markets. Market efficiency is also challenged if some simple investment strategy generates a return similar to that of the market, but at a systematically lower level of risk.

An interesting study in this regard is an empirical analysis of the characteristics of minimum-variance portfolios by Clarke, de Silva, and Thorley [2006] (CST). These authors find that minimum-variance portfolios based on the 1,000 largest U.S. stocks over the 1968-2005 period achieve a volatility reduction of about 25%, while delivering comparable or even higher average returns than the market portfolio.

We present a simple alternative approach to constructing portfolios with similar risk and return characteristics. Specifically, we create decile portfolios that are based on a straightforward ranking of stocks on their historical return volatility. Unlike CST, we effectively use only the diagonal of the historical covariance matrix with this approach. We find that portfolios of stocks with the lowest historical volatility are associated with Sharpe ratio improvements that are even greater than those documented in CST, and statistically significant positive alpha.

Ang et al. [2006] report that U.S. stocks with high volatility earned abnormally low returns over the 1963–2000 period. They focus on a very short-term (one-month) volatility measure, while we concentrate on long-term (over three years) volatility, which implies much lower

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is a senior researcher, Quantitative Strategies, at Robeco Asset Management. p.van.vliet@robeco.nl portfolio turnover. We find not only that high-risk stocks are exceptionally unattractive, but also that low-risk stocks are particularly attractive.

Ranking stocks on their historical volatility is related to ranking stocks on their historical capital asset pricing model betas. This follows theoretically from the fact that the beta of a stock is equal to its correlation with the market portfolio times its historical volatility divided by the volatility of the market portfolio. We also observe empirically that portfolios consisting of stocks with low volatility exhibit a low beta as well.

Since the earliest tests of the CAPM, researchers have shown that the empirical relation between risk and return is too flat (see, for example, Fama and MacBeth [1973]). Others such as Black, Jensen, and Scholes [1972] report that low-beta stocks have positive alpha. In their seminal research, Fama and French [1992] show that beta does not predict return in the 1963-1990 period, especially after controlling for size. In our sample, we also find alpha for portfolios ranked on beta, but considerably lower alphas than for portfolios ranked on volatility.

We contribute to the literature in several ways. First, we document a clear volatility effect: Low-risk stocks exhibit significantly higher risk-adjusted returns than the market portfolio, while high-risk stocks significantly underperform on a risk-adjusted basis.

Second, our findings are not restricted to the U.S. stock market, but apply to both global and regional stock markets. The alpha spread of the top-versus bottom-decile portfolio amounts to 12% per year for our universe of global large-cap stocks over the 1986-2006 period.

Third, we compare the volatility effect with the classic size, value, and momentum strategies, and control for these effects. In order to disentangle the volatility effect from the other effects, we use global and local Fama and French regressions and apply a double-sorting methodology. We find that the volatility effect is in fact a separate effect, and of comparable magnitude.

Fourth, we provide possible explanations for the success of the strategy that include leverage restrictions, inefficient industry practice, or behavioral biases among private investors, which all flatten the risk-return relation. Finally, we argue that benefiting from the low-volatility effect in reality is not easy if institutional investors do not include low-risk stocks as a separate asset class in their strategic asset allocation process.

DATA AND METHODOLOGY

At the end of every month, starting in December 1985 and ending in January 2006, we identify all constituents of the FTSE World Developed index, and take these as our universe for that particular month. This global large-cap universe consists of approximately 2,000 stocks on average; the actual number ranges between about 1,500 and 2,400 over time. Many return irregularities are known to disappear or become significantly less pronounced when the universe is restricted to large-caps, which makes our choice of universe conservative.

Our data sources are Factset for FTSE index constituents and return data, Compustat for U.S. fundamental data, Worldscope for non-U.S. fundamental data, and Thomson Financial Datastream for short-term interest rate data. Short-term interest rates are used to convert local stock returns into local stock returns in excess of the local risk-free return. Returns are log-transformed in order to make them additive over time. The log-transformed excess returns are used throughout our analysis for all return calculations.

At the end of each month, we construct equally weighted decile portfolios by ranking stocks on the past three-year volatility of weekly returns.² We also rank stocks on their book-to-market ratio (valuation), past 12-month minus 1-month total return (momentum), and free float market value (size). For the volatility and size measures, stocks with the lowest scores are assigned to the top decile, while for the valuation and momentum strategies stocks with the highest factor scores are the top decile.

Factor scores are compared directly across all stocks, without imposing sector or country restrictions. As a result, the entire Japanese market may be unattractive on valuation at the height of the Japan bubble during the late 1980s.

We do control for regional effects by presenting results for the U.S., Europe, and Japan in isolation. Portfolios are rebalanced monthly, and transaction costs are ignored throughout the analysis.

For each decile portfolio, we calculate the return (in excess of the local risk-free return) over the month following portfolio formation. For the resulting time series of returns, we calculate averages, standard deviations, and Sharpe ratios. To test for the statistical significance of the difference between two Sharpe ratios, we apply the Jobson and Korkie [1981] test with the Memmel [2003] correction.

EXHIBIT 1
Main Results—Global Decile Portfolios Based on Historical Volatility

	Panel A: Decile portfolios based on historical return volatility												
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10	Univ	
Excess return	7.3%	7.1%	8.0%	6.2%	5.0%	5.6%	5.6%	7.2%	3.4%	1.4%	5.9%	6.0%	
Standard deviation	10.1%	12.6%	13.6%	14.0%	14.0%	15.1%	16.5%	17.7%	20.4%	27.5%	23.7%	15.0%	
Sharpe ratio	0.72	0.57	0.59	0.44	0.36	0.37	0.34	0.41	0.17	0.05		0.40	
(t-value)	2.2	1.6	2.0	0.8	-0.6	-0.5	-1.1	0.2	-2.8	-2.7		-	
Beta	0.56	0.75	0.84	0.90	0.90	0.98	1.07	1.13	1.29	1.58	-1.02	1.00	
Alpha	4.0%	2.6%	3.0%	0.9%	-0.4%	-0.3%	-0.7%	0.4%	-4.3%	-8.0%	12.0%	-	
(t-value)	3.1	2.2	2.6	1.0	-0.4	-0.3	-1.0	0.4	-2.9	-2.6	3.0	-	
				Panel B: R	tisk analysi	s of portfol	ios based o	n historical	volatility				
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10	Univ	
Return up	-1.1%	-0.7%	-0.3%	-0.3%	-0.4%	-0.2%	0.2%	0.5%	0.6%	1.4%	-2.5%	0.0%	
Return down	1.8%	1.2%	0.9%	0.5%	0.4%	0.2%	-0.4%	-0.5%	-1.5%	-2.9%	4.8%	0.0%	
Max drawdown	-26%	-32%	-33%	-36%	-46%	-4 1%	-40%	-43%	-67%	-86%	-	-38%	
Max drawdown	-26%	-32%	-33%	-36%	-46%	-41%	-40%	-43%	-67/%	-86%	-	-38%	

This test statistic is calculated according to Equation (1) and asymptotically follows a standard normal distribution:

$$z = -\frac{SR_1 - SR_2}{\sqrt{\frac{1}{T} \left[2(1 - \rho_{1,2}) + \frac{1}{2} (SR_1^2 + SR_2^2 - SR_1SR_2[1 + \rho_{1,2}^2]) \right]}}$$
(1)

where SR_i refers to the Sharpe ratio of portfolio i, $\rho_{i,j}$ to the correlation between portfolios i and j, and T to the number of observations.

We use both a regression-based methodology and a double-sorting methodology in order to disentangle the volatility effect from other effects. Portfolios sorted on size and book-to-market are used to construct global and regional Fama-French equivalent hedge factors. We define small-minus-big (SMB) and high-minus-low book-to-market (HML) as the return difference between the top 30% and the bottom 30% ranked stocks. By regressing the return of volatility-sorted portfolios on these factors we control for possible systematic exposures to SMB and HML.

In a double-sorting routine, we first rank stocks on size or book-to-market and then on volatility within the size or book-to-market buckets. This is an empirically robust way to control for implicit loadings on these factors.

Fama-French adjusted alphas are estimated using the equation:

$$R_{i} = \alpha_{i} + \beta_{i}R_{m} + s_{i}SMB + h_{i}HML + \varepsilon_{i}$$
 (2)

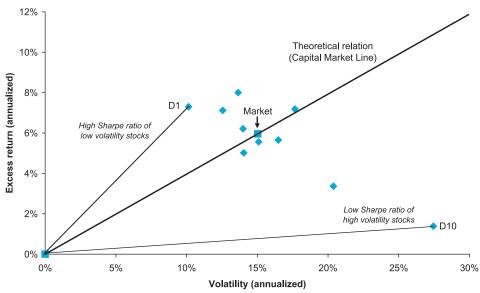
where R_i is the return on the decile portfolio i; α_i is the Fama-French adjusted alpha; R_m is the excess return on the global market portfolio defined as the equally weighted average of all stocks; and β_i , s_i , and h_i are the estimated factor exposures. Single-factor CAPM-adjusted alphas are calculated by including only the R_m factor in the regression. Statistical significance of the alphas is obtained in the usual manner.

GLOBAL RESULTS

Exhibit 1 provides an overview of the main results for the full global universe for decile portfolios ranked on past three-year volatility. The top-decile portfolio, which includes the low-risk stocks, can be seen to generate slightly above-average returns. In general, however, the relation between historical volatility and subsequent return appears to be rather weak, except for considerable underperformance in the bottom-decile portfolios (the high-risk stocks). The difference in average return between the top- and bottom-decile portfolio equals 590 basis points.

The results become more interesting when we shift to a risk-adjusted performance perspective instead of looking at straight returns. Ex post standard deviations increase monotonically for successive decile portfolios. The top-decile portfolio (D1) is about two-thirds as volatile as the market portfolio. Note that this volatility reduction is even greater than Clarke, de Silva, and Thorley [2006] find for U.S. minimum-variance portfolios.

EXHIBIT 2
Empirical Versus Theoretical Relation Between Volatility and Return



At the other end, we have the bottom-decile (D10) portfolio, with a standard deviation almost double that of the market portfolio. Combined with its low return, this results in a very low Sharpe ratio for the high-risk stock portfolio. Because the other decile portfolios exhibit relatively small differences in average returns, their Sharpe ratios are driven primarily by the standard deviation in the denominator.

One of our key findings is that the top decile of low-risk stocks achieves a Sharpe ratio of 0.72, compared to only 0.40 for the market portfolio. This difference in Sharpe ratios is statistically significant at the 5% level.

The Sharpe ratios show a steadily declining pattern across the volatility-sorted portfolios, and the Sharpe ratio of the bottom-decile portfolio is significantly lower (at the 5% level again) than the Sharpe ratio of the market portfolio. Thus, we observe a clear relation between ex ante volatility and ex post risk-adjusted returns. Exhibit 2 graphs these findings.

Exhibit 1's beta and alpha rows from a CAPM-style regression of monthly decile portfolio returns on monthly returns of the market show that the low-risk portfolio combines a very low beta of 0.56 with a positive alpha of 4.0% per year (statistically significantly different from zero at the 1% significance level). The betas increase monotonically for the successive decile portfolios, suggesting that volatility and beta are related risk measures. The

bottom-decile portfolio with the highest-risk stocks exhibits an estimated beta of 1.58 and a negative alpha of 8.0% per year. This finding implies a negative relation between risk and return. The combined alpha spread for the low-risk minus high-risk portfolio amounts to 12.0 percentage points.

Exhibit 3 illustrates these findings. The risk-return characteristics of the volatility-sorted portfolios clearly violate the theoretical (CAPM) security market line.

Panel B of Exhibit 1 displays additional characteristics of the volatility decile portfolios. The first two rows indicate the returns of the ten volatility portfolios with regard to up market versus down market months.

The low-risk portfolios underperform the market during up market months but outperform the market during down market months. This behavior is consistent with the low beta of the low-risk portfolios we have seen.

Note that underperformance during up months is considerably less than outperformance during down months, although this effect is countered to some degree by the more frequent occurrence of up months (59% up compared to 41% down months). The high-risk portfolios exhibit just the opposite behavior: outperformance during up months, but not by enough to offset the underperformance during down months.

The last row in Panel B shows maximum drawdown statistics, defined as the maximum loss that an investor in

these portfolios could have sustained (worst entry and worst exit moments). Just as the low-risk portfolios are only about two-thirds as volatile as the market, so are their maximum drawdowns, at 26% for the top-decile portfolio versus over 38% for the market. As one would expect, the greatest drawdowns (exceeding 80%) are experienced by the high-risk portfolios.

In Exhibit 4 we split the 20-year sample period into two 10-year subsamples. The low-volatility top-decile portfolios exhibit the highest Sharpe ratios in both subperiods. The alpha spread is significant in both the 1986-1995 and 1996-2005 periods. Nor does the effect appear to diminish over time, as levels and spreads of Sharpe ratios and alphas are higher during the more recent subperiod.

weight to be redistributed. Note that during more recent years the underweight of Japan has gradually disappeared.

At the sector level, the strategy tends to systematically overweight sectors such as utilities and real estate, while it usually avoids a typical high-risk sector such as information technology. For some other sectors, the positions taken by the strategy vary considerably over time. For example, the low-risk portfolio initially holds a significant number of telecom stocks. During the Tech bubble, stocks from the telecom sector are avoided, though, only to make a reappearance during the final years of the sample period.

RESULTS BY REGION

An inspection of the composition of the low-risk portfolio over time suggests a pronounced anti-bubble behavior. The strategy avoids the two main bubbles that occurred during our sample period: the Japan bubble in the late 1980s, and the Tech bubble in the late 1990s. Avoiding these bubbles initially results in underperformance, but once the bubbles burst, the low-risk portfolios tend to do particularly well.

The underweight of Japan is in fact the most significant country bet of the strategy, with the U.S. the primary beneficiary of the

E X H I B I T **3 Empirical Versus Theoretical Relation Between Beta and Return**

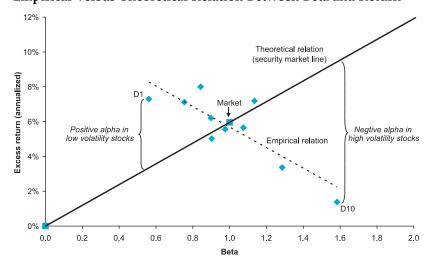


EXHIBIT 4
Subperiod Analysis of Global Decile Portfolios Based on Historical Volatility

Tallet A. Jan 1760 tillough Dec 1773													
D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-D10	Univ		
5.9%	6.1%	7.0%	4.4%	1.5%	3.0%	4.4%	5.4%	2.9%	1.7%	4.1%	4.4%		
11.9%	14.5%	15.5%	15.8%	14.8%	15.9%	17.5%	18.5%	19.3%	20.6%	13.7%	15.8%		
0.49	0.42	0.45	0.28	0.10	0.19	0.25	0.29	0.15	0.08		0.28		
2.9%	2.3%	2.9%	0.1%	-2.4%	-1.2%	-0.4%	0.4%	-2.2%	-3.6%	6.5%	-0.0%		
Panel B: Jan 1996 through Jan 2006													
D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-D10	Univ		
8.7%	8.1%	9.0%	8.0%	8.5%	8.1%	6.9%	9.0%	3.9%	1.0%	7.7%	7.5%		
8.0%	10.3%	11.5%	11.9%	13.3%	14.2%	15.4%	16.8%	21.5%	33.0%	30.6%	14.3%		
1.09	0.79	0.78	0.67	0.64	0.57	0.45	0.53	0.18	0.03		0.53		
5.7%	3.5%	3.6%	2.0%	1.7%	0.8%	-1.0%	0.4%	-6.9%	-14.2%	19.8%	-0.0%		
	5.9% 11.9% 0.49 2.9% D1 8.7% 8.0% 1.09	5.9% 6.1% 11.9% 14.5% 0.49 0.42 2.9% 2.3% D1 D2 8.7% 8.1% 8.0% 10.3% 1.09 0.79	5.9% 6.1% 7.0% 11.9% 14.5% 15.5% 0.49 0.42 0.45 2.9% 2.3% 2.9% D1 D2 D3 8.7% 8.1% 9.0% 8.0% 10.3% 11.5% 1.09 0.79 0.78	D1 D2 D3 D4 5.9% 6.1% 7.0% 4.4% 11.9% 14.5% 15.5% 15.8% 0.49 0.42 0.45 0.28 2.9% 2.3% 2.9% 0.1% Panel D1 D2 D3 D4 8.7% 8.1% 9.0% 8.0% 8.0% 10.3% 11.5% 11.9% 1.09 0.79 0.78 0.67	D1 D2 D3 D4 D5 5.9% 6.1% 7.0% 4.4% 1.5% 11.9% 14.5% 15.5% 15.8% 14.8% 0.49 0.42 0.45 0.28 0.10 2.9% 2.3% 2.9% 0.1% -2.4% Panel B: Jan 199 D1 D2 D3 D4 D5 8.7% 8.1% 9.0% 8.0% 8.5% 8.0% 10.3% 11.5% 11.9% 13.3% 1.09 0.79 0.78 0.67 0.64	D1 D2 D3 D4 D5 D6 5.9% 6.1% 7.0% 4.4% 1.5% 3.0% 11.9% 14.5% 15.5% 15.8% 14.8% 15.9% 0.49 0.42 0.45 0.28 0.10 0.19 2.9% 2.3% 2.9% 0.1% -2.4% -1.2% Panel B: Jan 1996 through J D1 D2 D3 D4 D5 D6 8.7% 8.1% 9.0% 8.0% 8.5% 8.1% 8.0% 10.3% 11.5% 11.9% 13.3% 14.2% 1.09 0.79 0.78 0.67 0.64 0.57	D1 D2 D3 D4 D5 D6 D7 5.9% 6.1% 7.0% 4.4% 1.5% 3.0% 4.4% 11.9% 14.5% 15.5% 15.8% 14.8% 15.9% 17.5% 0.49 0.42 0.45 0.28 0.10 0.19 0.25 2.9% 2.3% 2.9% 0.1% -2.4% -1.2% -0.4% Panel B: Jan 1996 through Jan 2006 D1 D2 D3 D4 D5 D6 D7 8.7% 8.1% 9.0% 8.0% 8.5% 8.1% 6.9% 8.0% 10.3% 11.5% 11.9% 13.3% 14.2% 15.4% 1.09 0.79 0.78 0.67 0.64 0.57 0.45	D1 D2 D3 D4 D5 D6 D7 D8 5.9% 6.1% 7.0% 4.4% 1.5% 3.0% 4.4% 5.4% 11.9% 14.5% 15.5% 15.8% 14.8% 15.9% 17.5% 18.5% 0.49 0.42 0.45 0.28 0.10 0.19 0.25 0.29 2.9% 2.3% 2.9% 0.1% -2.4% -1.2% -0.4% 0.4% Panel B: Jan 1996 through Jan 2006 D1 D2 D3 D4 D5 D6 D7 D8 8.7% 8.1% 9.0% 8.0% 8.5% 8.1% 6.9% 9.0% 8.0% 10.3% 11.5% 11.9% 13.3% 14.2% 15.4% 16.8% 1.09 0.79 0.78 0.67 0.64 0.57 0.45 0.53	D1 D2 D3 D4 D5 D6 D7 D8 D9 5.9% 6.1% 7.0% 4.4% 1.5% 3.0% 4.4% 5.4% 2.9% 11.9% 14.5% 15.5% 15.8% 14.8% 15.9% 17.5% 18.5% 19.3% 0.49 0.42 0.45 0.28 0.10 0.19 0.25 0.29 0.15 2.9% 2.3% 2.9% 0.1% -2.4% -1.2% -0.4% 0.4% -2.2% Panel B: Jan 1996 through Jan 2006 D1 D2 D3 D4 D5 D6 D7 D8 D9 8.7% 8.1% 9.0% 8.0% 8.5% 8.1% 6.9% 9.0% 3.9% 8.0% 10.3% 11.5% 11.9% 13.3% 14.2% 15.4% 16.8% 21.5% 1.09 0.79 0.78 0.67 0.64 0.57 0.45 0.53 0.18	D1 D2 D3 D4 D5 D6 D7 D8 D9 D10 5.9% 6.1% 7.0% 4.4% 1.5% 3.0% 4.4% 5.4% 2.9% 1.7% 11.9% 14.5% 15.5% 15.8% 14.8% 15.9% 17.5% 18.5% 19.3% 20.6% 0.49 0.42 0.45 0.28 0.10 0.19 0.25 0.29 0.15 0.08 2.9% 2.3% 2.9% 0.1% -2.4% -1.2% -0.4% 0.4% -2.2% -3.6% D1 D2 D3 D4 D5 D6 D7 D8 D9 D10 8.7% 8.1% 9.0% 8.0% 8.5% 8.1% 6.9% 9.0% 3.9% 1.0% 8.0% 10.3% 11.5% 11.9% 13.3% 14.2% 15.4% 16.8% 21.5% 33.0% 1.09 0.79 0.78 0.67 0.64 0.57 0.45 0.53 0.18 0.03	D1 D2 D3 D4 D5 D6 D7 D8 D9 D10 D1-D10 5.9% 6.1% 7.0% 4.4% 1.5% 3.0% 4.4% 5.4% 2.9% 1.7% 4.1% 11.9% 14.5% 15.5% 15.8% 14.8% 15.9% 17.5% 18.5% 19.3% 20.6% 13.7% 0.49 0.42 0.45 0.28 0.10 0.19 0.25 0.29 0.15 0.08 2.9% 2.3% 2.9% 0.1% -2.4% -1.2% -0.4% 0.4% -2.2% -3.6% 6.5% Panel B: Jan 1996 through Jan 2006 D1 D2 D3 D4 D5 D6 D7 D8 D9 D10 D1-D10 8.7% 8.1% 9.0% 8.0% 8.5% 8.1% 6.9% 9.0% 3.9% 1.0% 7.7% 8.0% 10.3% 11.5% 11.9% 13.3% 14.2% 15.4%		

Panel A: Jan 1986 through Dec 1995

EXHIBIT 5
Regional Results

						Panel A: M	ain results	US													
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10	Univ									
Excess return	6.9%	7.6%	8.1%	8.9%	8.3%	8.9%	9.6%	6.3%	7.0%	3.8%	3.1%	8.1%									
Standard deviation	12.0%	13.7%	15.4%	15.9%	17.0%	16.7%	18.0%	20.7%	25.8%	36.5%	35.0%	17.1%									
Sharpe ratio	0.58	0.56	0.53	0.56	0.49	0.53	0.53	0.30	0.27	0.10		0.47									
(t-value)	0.5	0.7	0.6	0.9	0.2	1.0	0.9	-2.6	-2.1	-2.6		-									
Beta	0.45	0.70	0.82	0.86	0.94	0.95	1.01	1.16	1.39	1.77	-1.32	1.00									
Alpha	3.3%	1.9%	1.5%	1.9%	0.7%	1.1%	1.4%	-3.2%	-4.3%	-10.6%	13.8%	0.0%									
(t-value)	1.6	1.3	1.1	1.4	0.5	1.3	1.3	-2.6	-1.9	-2.3	2.3	-									
					Pa	nel B: Mai	n results E	urope				Univ 4.9% 17.5% 0.28									
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10	Univ									
Excess return	6.0%	6.9%	6.3%	6.8%	5.2%	4.7%	3.5%	2.4%	3.7%	-0.0%	6.0%	4.9%									
Standard deviation	12.4%	14.2%	15.1%	16.9%	17.2%	17.8%	19.0%	20.2%	23.7%	28.7%	21.6%	17.5%									
Sharpe ratio	0.49	0.49	0.42	0.40	0.31	0.27	0.19	0.12	0.16	-0.00		0.28									
(t-value)	1.9	2.1	1.8	1.8	0.5	-0.2	-1.8	-2.7	-1.7	-2.7		-									
Beta	0.64	0.74	0.82	0.93	0.94	0.98	1.06	1.12	1.29	1.49	-0.85	1.00									
Alpha	2.9%	3.3%	2.4%	2.3%	0.7%	0.0%	-1.6%	-3.0%	-2.6%	-7.3%	10.2%	-0.0%									
(t-value)	2.4	2.6	2.1	2.1	0.6	0.0	-1.8	-2.9	-1.5	-2.7	2.9	-									
					P	anel C: Ma	in results J	apan													
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10	Univ									
Excess return	5.1%	5.1%	3.0%	4.6%	4.8%	4.2%	4.7%	3.5%	1.6%	-2.3%	7.5%	3.8%									
Standard deviation	15.2%	18.0%	19.6%	20.1%	21.5%	22.3%	23.2%	25.3%	27.1%	33.0%	25.5%	21.5%									
Sharpe ratio	0.34	0.28	0.15	0.23	0.22	0.19	0.20	0.14	0.06	-0.07		0.18									
(t-value)	1.3	1.3	-0.3	0.9	0.9	0.3	0.6	-0.7	-1.7	-2.6		_									
Beta	0.61	0.78	0.87	0.91	0.98	1.02	1.06	1.15	1.21	1.42	-0.81	1.00									
Alpha	2.8%	2.1%	-0.3%	1.2%	1.1%	0.4%	0.7%	-0.8%	-2.9%	-7.7%	10.5%	-0.0%									
(t-value)	1.6	1.5	-0.2	1.0	1.0	0.4	0.7	-0.7	-1.7	-2.8	2.5	_									

We perform a regional analysis in order to verify that the low-risk effect is not the result of some systematic regional bets. This analysis also sheds light on the robustness of the strategy. Panels A, B, and C of Exhibit 5 present the main results for the U.S., European, and Japanese markets in isolation, structured in the same way as Exhibit 1 for the global universe. The volatility effect turns out to be persistent over the three regions; the regional results are similar to the results on a global basis.

For all three regions there is not much evidence of anomalous behavior of the volatility portfolios if we take a simple return perspective, except for considerable underperformance of the bottom decile, i.e., the high-risk stocks. For the U.S. market, we even find that the top decile of low-risk stocks underperforms the market.

As in the global analysis, however, the picture changes dramatically if we take a risk-adjusted return perspective. All ex post standard deviations and betas increase monotonically for the successive volatility decile

portfolios. Within each region the top decile portfolios are only about 70% as volatile as the market. The bottom-decile portfolios are consistently at the other extreme, featuring standard deviations that are at least 50% to 100% higher than market standard deviations.

Combined with the very low returns of these portfolios, this results in Sharpe ratios that are negative or close to zero. On the other hand, the top-decile portfolios of low-risk stocks exhibit Sharpe ratios that are well above those of the market. The Sharpe ratio improvement is the greatest in Europe (a Sharpe ratio of over 0.49 for the low-risk portfolio versus 0.28 for the market), followed by Japan (0.34 versus 0.18), and then the U.S. (0.58 versus 0.47). In each region the Sharpe ratio of the high-risk bottom-decile portfolio is lower than that of the market at the 1% level of statistical significance.

The alpha spread is very consistent across the three main regions, varying from 10.2 percentage points for Europe to 13.8 for the U.S. The regional alpha spreads

E X H I B I T **6**Comparison with Other Investment Strategies

				Panel A: I	Decile portf	olios basec	on Size (n	narket capit	alization)						
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10	Univ			
Excess return	5.4%	7.6%	6.3%	6.7%	5.8%	5.5%	6.5%	5.6%	4.4%	4.5%	0.9%	6.0%			
Standard deviation	18.5%	15.6%	16.1%	15.9%	15.9%	15.8%	15.3%	14.7%	15.2%	15.2%	13.4%	15.0%			
Sharpe ratio	0.29	0.49	0.39	0.42	0.36	0.35	0.43	0.38	0.29	0.30		0.40			
(t-value)	-1.0	1.1	-0.1	0.6	-0.7	-1.0	0.5	-0.3	-1.3	-0.9		-			
Beta	1.10	0.98	1.03	1.03	1.04	1.03	0.98	0.95	0.95	0.90	0.20	1.00			
Alpha	-1.1%	1.7%	0.1%	0.6%	-0.4%	-0.6%	0.7%	-0.1%	-1.2%	-0.9%	-0.3%	-0.0%			
(t-value)	-0.6	1.5	0.1	0.8	-0.6	-0.9	0.8	-0.1	-1.0	-0.6	-0.1	-			
				Panel B	: Decile por	rtfolios bas	ed on Valu	e (book-to-	market)						
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10	Univ			
Excess return	9.3%	9.5%	8.8%	7.3%	4.8%	4.8%	3.8%	3.9%	4.0%	2.0%	7.3%	6.0%			
Standard deviation	20.0%	16.1%	15.0%	14.4%	15.0%	15.2%	15.0%	15.5%	15.9%	16.8%	14.4%	15.0%			
Sharpe ratio	0.46	0.59	0.59	0.50	0.32	0.32	0.25	0.25	0.25	0.12		0.40			
(t-value)	0.6	2.4	2.6	1.8	-1.5	-1.4	-2.6	-2.2	-1.8	-2.3		-			
Beta	1.20	1.02	0.95	0.93	0.97	0.98	0.98	0.99	0.99	0.98	0.22	1.00			
Alpha	2.1%	3.4%	3.1%	1.7%	-1.0%	-1.0%	-2.0%	-2.0%	-1.9%	-3.9%	6.0%	-0.0%			
(t-value)	1.1	3.1	3.3	2.2	-1.4	-1.3	-2.8	-2.2	-1.6	-2.1	1.9	-			
		Panel C: Decile portfolios based on Momentum (12-1M)													
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10	Univ			
Excess return	12.1%	8.6%	8.7%	6.1%	6.0%	5.1%	3.9%	2.5%	1.8%	0.9%	11.2%	5.9%			
Standard deviation	17.5%	14.9%	13.7%	13.7%	13.6%	14.0%	14.5%	16.4%	19.4%	27.0%	24.0%	15.0%			
Sharpe ratio	0.69	0.58	0.63	0.44	0.44	0.36	0.27	0.15	0.09	0.03		0.39			
(t-value)	2.0	1.7	2.5	0.7	0.9	-0.5	-2.1	-3.1	-3.0	-2.5		_			
Beta	0.96	0.89	0.85	0.87	0.88	0.90	0.94	1.05	1.19	1.49	-0.53	1.00			
Alpha	6.5%	3.4%	3.7%	1.0%	0.9%	-0.2%	-1.6%	-3.6%	-5.2%	-7.9%	14.3%	-0.0%			
(t-value)	2.9	2.3	3.3	1.0	1.2	-0.3	-2.1	-3.4	-3.1	-2.3	2.8	_			

are of comparable magnitude as the alpha spread at the global level (12.0 percentage points), which implies that bottom-up regional allocation is not the key driver for the global results. The alpha spreads are statistically significantly different from zero at the 5% level for the U.S. and Japan and at the 1% level for Europe.

CONTROLLING FOR OTHER EFFECTS

How does the volatility effect relate to other effects that have been documented in previous research? For example, could it be that the low-volatility portfolio holds a high proportion of value stocks, and that as a result it is simply capturing the value premium? And how does the extent of the volatility effect compare to classic effects such as value, size, and momentum?

To help answer these questions, Exhibit 6 displays the same statistics as for the low-volatility portfolios, but now for the classic value, momentum, and size strategies. Like other researchers, we find that the top deciles of the value and momentum strategies outperform the equally weighted universe, while the bottom deciles underperform. There is little evidence of a size effect in our sample of FTSE World Developed index constituents.

Interestingly, the low-volatility top-decile portfolio delivers a higher return per unit of risk (Sharpe ratio) than each individual value, momentum, or size decile portfolio. From an alpha perspective, the volatility effect ranks second of four; only the momentum effect is somewhat stronger in our sample. Given this analysis, we conclude that the volatility effect holds up well in terms of impact and thus economic relevance in comparison to other classic effects.

A comparison of the characteristics of the volatility decile portfolios and the other decile portfolios suggests that the low-volatility effect does indeed constitute a separate effect. For example, the top-decile portfolios on

EXHIBIT 7
Global and Regional Fama-French Corrected Alphas

				I	anel A: Fan	na-French co	orrected alph	as			
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10
Global Alpha	4.0%	2.6%	3.0%	0.9%	-0.4%	-0.3%	-0.7%	0.4%	-4.3%	-8.0%	12.0%
FF-Alpha	2.8%	1.3%	1.8%	0.3%	-0.8%	-0.3%	-0.6%	1.2%	-2.9%	-5.4%	8.1%
				Panel	B: Regiona	l Fama-Fren	ch corrected	alphas			
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10
US	3.3%	1.9%	1.5%	1.9%	0.7%	1.1%	1.4%	-3.2%	-4.3%	-10.6%	13.8%
FF-Alpha	1.3%	0.6%	0.2%	1.0%	0.3%	0.8%	1.0%	-2.8%	-2.9%	-5.7%	7.0%
Europe	2.9%	3.3%	2.4%	2.3%	0.7%	0.0%	-1.6%	-3.0%	-2.6%	-7.3%	10.2%
FF-Alpha	3.2%	3.0%	1.9%	1.9%	0.3%	-0.5%	-2.3%	-3.8%	-2.1%	-4.2%	7.4%
Japan	2.8%	2.1%	-0.3%	1.2%	1.1%	0.4%	0.7%	-0.8%	-2.9%	-7.7%	10.5%
FF-Alpha	2.1%	1.9%	0.1%	1.2%	0.5%	0.0%	0.7%	-0.5%	-1.7%	-7.7%	9.8%
FF-Alpha	2.1%	1.9%	0.1%	1.2%	0.5%	0.0%	0.7%	-0.5%	-1.7%	-7.7%	9.8%

value, size, and momentum exhibit higher volatility than the market, while the low-volatility portfolio is only about two-thirds as volatile as the market. Also, the betas of the value, size, and momentum top-decile portfolios are close to, or even above 1.00, while the low-volatility top-decile portfolio has a beta of only 0.56. These very different characteristics suggest that the low-volatility effect is a distinct effect, and not some classic effect in disguise.

Exhibit 7 further differentiates the volatility effect from the other effects by means of Fama-French (FF) regressions. Panel A shows the alphas corrected for value and size using a global Fama-French factor model. We find that one-third of the global alpha spread of 12.0 percentage points can be attributed to size and value exposures. The 8.1% of alpha that remains is thus not related to value and/or size and is left unexplained.

Panel B shows the results of similar analyses at the regional level, based on local Fama-French regressions. The FF adjustment has the greatest impact for the U.S., where the alpha drops from 13.8% to 7.0%. For Europe, the alpha is lowered from 10.2% to 7.4%. The alpha is least affected for Japan, at 9.8% versus 10.5%.

According to these results, we can conclude that the volatility effect is reduced, but does not disappear after applying the FF adjustment. The FF adjustment is based on a single regression, which is applied ex post to the time series of returns. Thus, the factor exposures are estimated and assumed to be constant over time.

An alternative way to disentangle the volatility effect from other cross-sectional effects is to apply a doublesorting approach. This is a robust non-parametric technique that enables us to systematically neutralize other effects ex ante.

Panel A of Exhibit 8 shows the results of a double sort on value followed by volatility. Every month stocks are first grouped into five quintiles based on value (bookto-market). Next we create decile portfolios on the basis of volatility within each of these value quintiles. Finally, we construct a value-neutral top-decile volatility portfolios from within each value quintile (and similarly for the other decile portfolios). Panel B provides similar results for a double sort on size followed by volatility, and Panel C for a double sort on momentum followed by volatility.

The volatility effect turns out to be robust to the ex ante factor neutralizations of the double sorts. The global (CAPM-)alpha remains at 8.9% or higher, and the alpha for the U.S. at 9.5% or higher. For Europe we find that the alpha drops to 6.4%, in case of the momentum double sort, and for Japan to 6.6%, in case of the value double sort. Again we conclude that classic effects at most explain only part of the volatility effect.

ROBUSTNESS TESTS

The volatility effect is robust to a different measurement period for volatility. Panel A of Exhibit 9 shows the CAPM alphas of decile portfolios based on one-year instead of three-year weekly historical return volatility. The top- versus bottom-decile alpha spread is slightly lower in a global context (11.2 versus 12.0 percentage points), but can be seen to increase somewhat for both the

EXHIBIT 8

Double-Sorted Results

			Panel A:	Alpha from d	ouble sort on	value (book-t	o-market) an	d volatility (p	ast 3 years)				
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10		
Global	3.5%	3.0%	2.5%	0.6%	0.4%	-0.1%	-1.1%	-0.6%	-3.1%	-7.6%	11.1%		
US	3.4%	1.7%	2.3%	2.4%	1.2%	-0.5%	0.3%	-0.9%	-5.7%	-9.4%	12.7%		
Europe	2.4%	3.7%	2.7%	0.9%	0.7%	0.1%	-0.6%	-3.0%	-2.1%	-7.3%	9.7%		
Japan	1.3%	0.5%	0.5%	0.8%	1.0%	0.2%	-0.8%	0.7%	-1.8%	-5.3%	6.6%		
Panel B: Alpha from double sort on size (market capitalization) and volatility (past 3 years)													
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10		
Global	3.9%	2.4%	2.6%	1.6%	0.4%	-1.5%	0.7%	-1.6%	-2.9%	-8.3%	12.2%		
US	3.8%	1.6%	1.6%	2.3%	1.0%	1.3%	-0.7%	-2.6%	-6.0%	-7.8%	11.6%		
Europe	2.7%	3.5%	1.4%	0.9%	2.6%	-1.4%	-1.2%	-1.0%	-3.4%	-6.5%	9.2%		
Japan	1.5%	2.4%	0.3%	1.6%	0.2%	0.9%	-0.2%	0.5%	-1.0%	-8.7%	10.2%		
			Panel C	: Alpha from	double sort o	n momentum	(12-1M) and	volatility (pas	st 3 years)				
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10		
Global	2.9%	2.5%	1.5%	0.7%	0.3%	0.3%	-1.1%	-0.5%	-2.0%	-6.0%	8.9%		
US	3.0%	2.0%	1.8%	1.6%	-0.6%	0.5%	-0.3%	-0.9%	-4.2%	-6.5%	9.5%		
Europe	1.7%	3.0%	1.3%	0.2%	0.7%	1.1%	-1.3%	-1.2%	-2.5%	-4.7%	6.4%		
Japan	2.8%	1.3%	0.8%	0.7%	1.0%	0.7%	0.4%	-0.7%	-2.3%	-7.2%	10.1%		

EXHIBIT 9
Robustness Tests

	Panel A: CAPM-alpha for regional decile portfolios sorted on 1 year (instead of 3 year) volatility												
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10		
Global	3.1%	2.0%	1.8%	1.4%	-1.1%	-0.5%	-0.3%	0.1%	-1.4%	-8.1%	11.2%		
US	4.1%	0.3%	2.3%	2.0%	1.5%	1.6%	1.8%	-4.0%	-5.3%	-10.4%	14.5%		
Europe	3.0%	2.7%	1.3%	1.8%	0.7%	0.0%	-0.7%	-1.5%	-1.7%	-8.7%	11.7%		
Japan	0.8%	1.3%	2.1%	0.2%	1.3%	0.2%	-0.5%	-0.2%	-2.6%	-6.3%	7.1%		
Panel B: CAPM-alpha for regional decile portfolios sorted on volatility versus beta													
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10		
US volatility	3.3%	1.9%	1.5%	1.9%	0.7%	1.1%	1.4%	-3.2%	-4.3%	-10.6%	13.8%		
US beta	1.2%	1.7%	1.8%	2.5%	0.2%	1.5%	1.2%	-3.2%	-4.2%	-8.1%	9.3%		
Europe volatility	2.9%	3.3%	2.4%	2.3%	0.7%	0.0%	-1.6%	-3.0%	-2.6%	-7.3%	10.2%		
Europe beta	1.5%	2.4%	2.8%	0.9%	1.4%	0.7%	-0.7%	-1.8%	-4.1%	-5.9%	7.5%		
Japan volatility	2.8%	2.1%	-0.3%	1.2%	1.1%	0.4%	0.7%	-0.8%	-2.9%	-7.7%	10.5%		
Japan beta	-1.4%	1.2%	1.1%	1.5%	0.7%	1.3%	1.3%	0.7%	-5.3%	-5.3%	3.8%		
		I	Panel C: Alp	ha from do	ouble sort or	n beta (past	3 years) an	d volatility	(past 3 yea	rs)			
	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D1-10		
US	2.3%	0.8%	0.8%	0.6%	0.6%	-0.5%	0.2%	-2.9%	-2.4%	-2.1%	4.4%		
Europe	2.3%	1.4%	1.5%	1.5%	0.0%	0.7%	-1.3%	-0.2%	-2.9%	-4.8%	7.1%		
Japan	2.0%	1.8%	1.6%	0.1%	-1.1%	1.0%	-0.1%	-0.3%	-2.1%	-4.9%	6.9%		

U.S. and Europe. Only for Japan does the alpha spread drop by a relatively large amount, from 10.5 to 7.1 percentage points.

We also compare the volatility effect with the classic beta effect. Unlike volatilities, estimated betas are sensitive to the choice of the market portfolio. Beta can be estimated relative to a global index, but also relative to a regional index. This is an important empirical issue. For example, the Japanese market has shown a low correlation with the global stock market, which, ceteris paribus, results in lower estimated betas relative to a global index for all Japanese stocks.

To avoid this effect, and to produce comparable results with other (U.S.) studies, we concentrate on betasorted portfolios at the regional level.

In Panel B of Exhibit 9 we compare the CAPM alphas of portfolios sorted on three-year historical volatility and three-year historical beta, again calculated using weekly return data. For each region we find a clear beta effect, but the alpha spreads of the beta-sorted portfolios are about 3% to 7% lower for each region, and the alpha patterns are more irregular than those for the volatility-sorted portfolios. Therefore, we conclude that the volatility effect is a stronger and less ambiguously defined effect than the beta effect.

Further evidence supporting this conclusion is shown in Panel C of Exhibit 9, which gives results of double-sorting first on beta and then on volatility. Although this way the alpha is partly subsumed, about 7% remains for Europe and Japan and 4% for the U.S. Thus, even within groups of stocks with similar betas, sorting stocks on volatility helps to capture additional alpha. Thus, the volatility effect cannot be explained by the classic beta effect. This finding suggests moreover that both the idiosyncratic part and the systematic part of volatility are mispriced.

POSSIBLE EXPLANATIONS

There are several possible explanations for the volatility effect that we have documented.

First, leverage is needed in order to take full advantage of the attractive absolute returns of low-risk stocks. In theory this is quite straightforward, but in practice many investors are either not allowed or are unwilling to actually apply leverage, especially on the scale needed to take advantage of this effect. For example, if a low-risk stock portfolio is two-thirds as volatile as the market, 50% leverage needs to be applied in order to obtain the same level of volatility as the market. As a result, the opportunity presented by low-risk stocks is not easily arbitraged away. Borrowing restrictions were long ago held up by Black [1972] as a reason for the relatively good performance of low-beta stocks.

Leveraged buyout (LBO) private equity funds might constitute a notable exception in this regard, because a key source of return of LBO funds is the application of leverage to the balance sheets of their portfolio companies. Thus, the success of LBO private equity investing may, to some degree, be related to the high risk-adjusted returns of low-risk stocks. Pure equity investors may face

practical limitations with regard to leverage, but we want to stress that leverage can be created relatively easily within a balanced portfolio that holds bonds and cash along with stocks.

Black [1993] some time ago suggested an increased allocation to low-risk stocks as an alternative to a given allocation to the market portfolio. That is, instead of investing 50% in traditional stocks and 50% in bonds, an investor might decide to invest 70% in low-risk stocks and 30% in bonds. Of course, this requires that low-risk stocks be included as a separate asset class in the strategic asset allocation process of investors. This is not the case in practice—at least not yet.

Second, the volatility effect could be the result of an inefficient decentralized investment approach. For example, in the professional investment industry it is common practice that first the chief investment officer or an investment committee makes the asset allocation decision, and in a second stage capital is allocated to managers who buy securities within the different asset classes. Van Binsbergen, Koijen, and Brandt [2007] demonstrate that this approach may result in inefficient portfolios.

The problem with benchmark-driven investing is that asset managers have an incentive to tilt toward highbeta or high-volatility stocks, as this is a relatively simple way for asset managers to generate above-average returns, assuming the CAPM holds at least partially. As a result, these high-risk stocks may become overpriced, while low-risk stocks may become underpriced, which is consistent with the return patterns that we document. Furthermore, new money tends to flow toward asset classes that do well, and within such asset classes to managers with above-average performance.

This suggests that for a profit-maximizing asset manager outperformance in up markets may be more desirable than outperformance in down markets. Asset managers may thus be willing to overpay for stocks that outperform in up markets, which tend to be high-volatility stocks, and underpay for stocks that outperform in down markets, which tend to be low-volatility stocks.

Basically, asset managers' twin desire for outperformance and cash flow may result in inefficient portfolios. A solution may be to integrate the two-stage process by giving asset managers one single benchmark, such as fund-specific liabilities, plus a risk budget to deviate from that.

Finally, the volatility effect may be caused by behavioral biases among private investors. Behavioral portfolio theory postulates that private investors think in terms of

a two-layer portfolio. Shefrin and Statman [2000] identify a low aspiration layer, which is designed to avoid poverty, and a high aspiration layer, which is designed for a shot at riches. Suppose private investors make a rational risk-averse choice in the asset allocation decision (first layer), but become risk-neutral or even risk-seeking within a certain specific asset class (second layer). In this case, investors will overpay for risky stocks, perceived to be similar to lottery tickets.

From this perspective, buying many stocks destroys upside potential, while buying a few volatile stocks (like Microsoft in the 1980s) leaves upside potential intact. This way of thinking is consistent with the finding that most private investors hold only about one stock to five stocks in their portfolios, thereby largely ignoring the

diversification benefits that are available in the equity market. Investor divergence from risk-averse behavior may also cause high-risk stocks to be overpriced and lowrisk stocks to be underpriced.

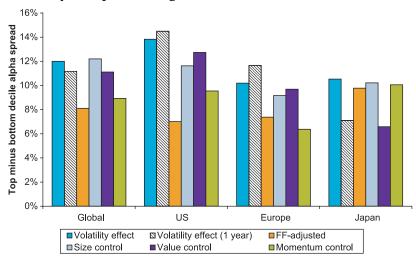
CONCLUSION AND IMPLICATIONS

We have shown that stocks with low historical volatility have superior risk-adjusted returns, both in terms of Sharpe ratios and in terms of CAPM alphas. The volatility effect is similar in size to classic effects such as value, size, and momentum, and largely remains after Fama-French adjustments and double-sorts.

Our major results are summarized in the bar chart in Exhibit 10. While Clarke, de Silva, and Thorley [2006] find significantly lower risk and higher Sharpe ratios for U.S. minimum-variance portfolios, our results are stronger, and our approach is easier. Our results are consistent with Ang et al. [2006], who document a large negative alpha for U.S. stocks with high idiosyncratic volatility, but our results are more symmetric, as well as based on three-year instead of one-month historical volatility, which implies a much lower portfolio turnover.

The volatility effect is particularly strong in a global setting, with a low- versus high-volatility alpha spread of 12 percentage points. The results remain strong, however, at the regional level (>10 percentage points). The low-volatility strategy is characterized by relatively small drawdowns, a low beta, outperformance in down mar-

EXHIBIT 10
Summary of Alpha Findings



kets and underperformance in up markets, and anti-bubble behavior. Possible explanations for the success of the strategy include: 1) the practical difficulties of arbitraging the effect away with significant leverage, 2) inefficient industry practice, or 3) behavioral biases among private investors. All these elements flatten the risk-return relation.

Exploiting the volatility effect is not easy for benchmark-driven equity investors who are facing a relative return objective and either not allowed to or willing to apply leverage. For investors interested in high-Sharpe ratio investment opportunities such as pension funds, however, it may be much easier to benefit from the volatility effect, if they can apply leverage in their asset mix. These investors could simply decide to shift from a given allocation to traditional stocks to a higher allocation to low-risk stocks by reducing the weight of bonds.

In order to take this option into account effectively, it is essential to include the decision to invest in low-risk stocks in the strategic asset allocation process. Therefore, we recommend that absolute- return investors differentiate low-risk, high-risk, and traditional stocks as separate asset classes, just as they distinguish between value and growth stocks and large-cap and small-cap stocks in their strategic asset allocation decision-making.

ENDNOTES

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¹Note that using these returns is equivalent to assuming that first all currency risk is hedged to the particular base currency, and next converting these currency-hedged stock returns to excess returns, by subtracting the risk-free return of the particular base currency.

²All results presented are based on equally weighted portfolios. For cap-weighted portfolios we find similar results, but they are not presented for the sake of brevity.

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